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Letter from the Editors

We are very excited to be the new co-editors of the Oklahoma Journal of School Mathematics, the official publication of the Oklahoma Council of Teachers of Mathematics. We would like to thank the previous authors, Juliana Utley and Stacy Reeder, for their guidance and support as we embark on this journey.

There are several ways that you can contribute to the school mathematics community through the Oklahoma Journal of School Mathematics. One way is by submitting a manuscript. We want to hear about the successful things you are doing in your classrooms. OkJSM is currently accepting articles that focus on teacher research and activities you have used in your classroom. Have you read some good research or found a professional resources that you want to share? There is also a place for reviews in the journal. Our membership includes teachers of students from pre-k to the university level, so we need articles for a wide range of grade levels, content areas, and topics.

Another way to contribute to by volunteering to review manuscripts. We need a reviewers with experience in various grade bands, including early childhood, elementary, middle school, high school, university mathematics, and teacher education. We also need reviewers with expertise and special interests in topics such as assessment, cognition/brain research, curriculum, math integration, pedagogy, social justice/equity, and special education. We are asking all who are interested in reviewing (including those who have reviewed in the past) to fill out an application at https://bit.ly/OkJSMReviewer.

Finally, you can contribute by becoming a regular reader of the journal. Our plan is to start by publishing OkJSM once a year. (We are looking at other publishing opportunities for those who present at the annual conference.) When you see the journal has been published, we ask that you take the time to read the articles, reflect how you can use them, and share with others.

We look forward to sharing the great work that is happening in math classrooms!

Darlinda Cassel — Jill M. Davis

Darlinda Cassel — Jill Davis

Integrating Writing in a Mathematics Classroom

By Amanda G. Myers

Writing is an important component of educating students and has been important for many years. Writing promotes students' thinking toward a deeper understanding of the subject matter and provides an avenue for students to express their thoughts to their peers. It provides the opportunity for students to build communication skills as well. Students are learning how to speak and listen when they are given the opportunity to share their thoughts in writing and allowed to share their ideas with others. Writing experiences have been strongly provided for students in reading and language arts classrooms. Reading and language arts classes offer an environment where writing is easily integrated and encouraged. It makes sense for students to be writing in these classes. Science and social studies classes present students the chance to research events in order to write an essay, write biographies, and conduct science experiments that include writing procedures, hypotheses, and conclusions. However, observing students writing in a mathematics classroom is a less likely occurrence. Writing in a mathematics classroom supports learning because it involves students organizing and reproducing their thoughts and ideas, which is useful in making sense of mathematics. Writing allows students to express their understanding, misunderstandings, and feelings toward the mathematics content being taught (Burns, 2004). No longer are students being asked to memorize facts or use algorithms to solve problems. Students are now being required to demonstrate their mathematical understanding and mathematical thinking by expressing their thoughts and ideas using writing techniques. Kostos and Shin (2010) state "mathematical thinking and mathematical communication go hand-in-hand" (p. 223). Students' focus has to change from simply being able to get the correct answer to being able to explain the method or strategy used to get that correct answer (Kostos & Shin, 2010). The writing students engage in during mathematics is not intended to be published but rather a way for students to organize their thinking and explore the mathematics content in a way that produces a deep mathematical understanding (Burns, 2004). According to Bosse and Faulconer (2008), writing deepens mathematical understanding and extends student thinking which extends their mathematical learning of content knowledge.

What is the most effective and most used form of writing in a mathematics classroom? Research has shown that the most effective form of writing in a mathematics classroom is the use of journals. Journals are notebooks that students use to respond to a teacher's question or reflect on the teacher's lesson. All responses are an expressive style of writing and are spontaneous. Students' writing is written in paragraph form using sentences rather than numbers and equations. According to Nahrgang and Petersen (1986), journals have two purposes. Journals assist students with gaining mathematical understanding and provide teachers with an assessment tool. Journals allow students to proceed at their own pace. Providing students the opportunity to work at their own pace allows students to develop an understanding of mathematical concepts based on their own prior knowledge and experiences. Journals convey students' confusion and misunderstandings that can be addressed by the teacher or the journal entries will reveal a students' deep understanding of the concepts that the teacher can build upon. Students' reflections help guide further instruction by the teacher.

Journal writing allows students to be engaged in active learning. Based on the findings by Miller (1992), students may lose focus or develop lack of interest if they are simply listening to the teacher's lesson or copying notes the teacher has written on the board. Allowing students to use journal writing during the lesson for recording misunderstandings or for posing questions that they need to be answered will assist them in making sense of the concept which is more beneficial than simply copying the teacher's notes. This allows students to be active learners and keeps students focused on the lesson. Students can express their ideas and thoughts about the lesson as it is being taught. Journal writing during the lesson can be used for note taking that is conducted by the students and should be kept personal. These entries are for the student to use as a reference or guide to help make sense of mathematics concepts. Journal writing should be risk free and teachers should emphasize the importance of honest entries. Honest entries provide correct information about students' understandings which allows the teacher to better instruct the lessons. Once students understand the expectations of the journal entries, students will start to write freely with more expression of their thoughts and ideas. These entries can be used to ask teachers questions or concerns students may have, or it provides a way for students to contribute to the discussion. When students are active participates in the class discussion, it helps students build their communication skills and mathematical content knowledge. Journals can be incorporated into the use of an interactive notebook. The interactive notebook allows students to create foldables to help develop vocabulary and provides a reference guide for students to refer to as needed.

Journals can be used at the end of the lesson in order for students to reflect on the lesson previously taught. Students can express their thoughts and ideas about the lesson that has been presented, which will help students build a better understanding of the content. These journal entries can be kept personal or shared with teacher and peers. Sharing thoughts or questions about the previous lesson will help clarify the lesson for the individual student sharing, but it also helps the students listening to make sense of the mathematical content. Students may communicate feelings and emotions about a subject. Students can also convey what they liked or did not like about the lesson or unit (Borasi & Rose, 1989). This information guides the teacher in planning future lessons that are meaningful. The interactive notebook can allow students to respond to a question related to the topic of the day.

The use of writing in a mathematics classroom provides several important benefits for our students which is the main purpose for having them write about mathematics. However, teachers gain from students' writing in their classrooms as well. According to Miller (1992), teachers have the opportunity to build on their instructional practices. Teachers are able to assess their teaching by having students respond to lessons being taught. Students are allowed the freedom to express their successes and misconceptions of a lesson which provides meaningful information to a teacher about how different students respond to the same lesson. This information allows the teacher to reteach if necessary, delay exams, review, and provide individual discussions and conferencing. The delivery of the lesson or instructions may have been confusing for students which can be revised as long as students are allowed to express these thoughts in their journal writing. When students and teachers develop conversations about mathematics, respect and a relationship is developed. Developing relationships with students helps learning become important and builds mathematical growth.

Research has shown writings produced in a mathematics classroom will become an assessment tool for teachers. Teachers learn which students have an understanding of the content being taught as well as the level of understanding. Teachers identify students' strengths and weaknesses which can be used to prepare for new activities that are differentiated. Journal writings provide a quick and personal way for teachers to provide unique feedback. A journal establishes a meaningful conversation between students and teacher and provides a means of communication that may not ever take place due to time constraints in the classroom or due to students' lack of confidence. Journals offer an approach for students and teachers to create a community environment that is built on mutual respect. Developing mutual respect will build self-confidence which will help build on students' content knowledge. It is important for students and teachers to develop a relationship that increases mathematical learning.

What are other forms of writing that can be utilized in a mathematics classroom? Students can be asked to create a story about a mathematical situation. Students can be given a mathematical equation in which they use the equation to invent the text. Real-life experiences can be used to provide inspiration for the created stories. Students can begin by using manipulatives to help develop stories with mathematical meanings. When given this task for the first time students may begin with only a few sentences. But after many opportunities to create story problems, students will start to draw from experiences and invent exciting stories. Essays about mathematical content may be an extension to writing text for an equation. Mathematics content can be clarified and developed when writings are about a specific topic. Essays provide writing, consulting, revising, and rewriting opportunities for students which develop writing skills as well as a deeper understanding of mathematics content. Essays also provide useful information for assessing what is understood and misconceptions students still have about a specific content area (Stempien & Borasi, 1985).

Shobert (2014), provides a variety of writing activities that can be incorporated into a mathematics classroom. Cartoons are a creative and fun way for students to express mathematical ideas. They can create their own cartoon using paper or use technology to create an animated video. Cartoons can allow students the opportunity to explain how they solved a problem by being inventive and creative. Exit slips are a quick and simple way for teachers to assess students. Students can be asked to answer a question or provide an explanation of a concept they learned that day. This can be their ticket out of the classroom. Another writing strategy is having the students explain concepts. Students can be asked to explain a concept so that an absent student will understand. Students will have to decide the most important points and be able to organize their thinking. Error analysis is a way to get students thinking about mathematics. Students can be given a problem with a mistake and provide an explanation of the mistake and the reason the mistake was made. Students can provide an appropriate solution and the reason it works. Students can develop an explanation for the mistake and suggestions for correcting the mistake.

Posting useful mathematics vocabulary that arises during class discussions provides a reference for students' writings. Prompts are an appropriate way to begin integrating writing. The prompt can relate to the lesson or assignment and students can discuss their ideas before writing. Prompts allow students to include details in their explaining which promotes a thorough piece of writing. Students need to understand the importance of writing about mathematics. They should be given the opportunity to express their thoughts and ideas personally, and then be allowed to share their writing with a partner, small group, or whole class if comfortable. The sharing of students' writing provides feedback to the author but also enables students to hear other points of view. Students' writings can provide ideas for extending the lesson or reinforcing content already taught. Teachers need to use students' writing to reflect on previous lessons to determine if the lesson had meaning, student understanding, interest, or if it was challenging for students. Students' writing can help guide the following class discussions. Students' writings can be kept in individual folders. This provides a chronological collection of work for each student and provides a useful record of students' progress. The folders of work are a beneficial tool to share with parents at conferences.

There are many forms of writing that can be incorporated into a mathematics classroom. Students can use glogster.com to create a virtual poster combining text, audio, video, images, and hyperlinks to share with others electronically. Students can create plays, songs, write biographies of mathematicians, and write about jobs that use mathematics and describe how mathematics is used in the real world. They can take pictures of math being demonstrated on the playground and write a description of how it is being used and they can write stories using the academic vocabulary. At the end of the lesson, students can be given the opportunity to write on a sticky note, "What Stuck with You" and post the note on a bulletin board with this title.

Writing in the mathematics classroom provides many benefits to students and teachers. Students' and teachers' attitudes increased during an experimental writing period in a mathematics class. Both wanted to continue spending time writing in their mathematics classes (Miller, 1992). Mett (1989), also discovered students had an appreciation for writing as well as an appreciation for mathematics. Jurdaka and Zein (1998), found journal writing was not only rewarding to students but to parents as well. Parents visited the school to express their support for journal writing and asked for it to be a continuation to the program. The relationship between students and teacher changed to a more positive and respectful relationship. Not only did the relationship change to the positive, so did the atmosphere of the classroom (Borasi & Rose, 1989). Students' confidence in writing and mathematics increased (Clarke, Waywood, & Stephens, 1993). The articles and research reinforces the idea that writing in the mathematics classrooms has many important aspects. Writing provides students the opportunity to reflect on mathematics content. Students are allowed to summarize the lesson and share thoughts and ideas with peers. Writing permits students a way to organize their thinking before they are expected to share with their peers their reflection of the content. Sharing thoughts and ideas generates rich vocabulary that is evident by the meaningful class conversations and discussions. The ability to communicate ideas to others is important for any future endeavor of a student. Writing helps students develop a deep understanding of mathematics content. Students are expressing their understanding or misconceptions they may have about the content. This is a benefit to teachers as well. Teachers are able to assess their teaching based on students' writing. They may need to reteach a lesson, review, extend a lesson, or provide individual conferencing. Teachers identify students' strengths and weaknesses which can guide future instruction. Teacher and students develop a relationship of mutual respect when students are given the opportunity to express themselves. The only negative aspect that was recorded by Kostas and Shin (2010), was the time constraint. Teachers were concerned about the time it would take for students to write during class time. Teachers felt it would take away from the mathematics content that needed to be taught. Teachers were also concerned about how much time it would take for them to respond to students' writing. Time is not a significant or legitimate reason to abandon the idea of writing in a mathematics classroom. I believe the benefits to writing in a mathematics class outweigh the concern about time. There are many positive reasons for allowing students to write in a mathematics classroom. Students are learning mathematics content, writing skills, and communication skills. Students learn to organize their mathematical thinking, communicate their mathematical thinking to peers and teachers, and develop vocabulary through mathematical expression.

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STEM: Applications in the Classroom

By Dan Vincent and Darlinda Cassel

With the recent emphasis on STEM education in K-12 schools, teachers may find themselves wondering how to integrate these four areas in an authentic, engaging way. We have found it helpful to first understand the goals / purposes behind two distinct content areas, science and engineering, and how to appropriately use technology and mathematics within those areas.

To differentiate science and engineering, the first place to begin is looking at the purpose behind each. As stated in the Next Generation Science Standards "8 Scientific and Engineering Practices" (NGSS Lead States, 2013), science is focused on posing problems and constructing explanations, whereas engineering is focused on defining problems and designing solutions (the other six 'Practices' do not differentiate science from engineering). Although the differences are subtle, the way tasks are given to students can vastly change how students work through given tasks (see Harkema, Jadrich and Bruxvoort, 2009). For example, take a common elementary activity of having students investigate the way a toy car moves down a ramp by modifying the angle of the ramp. To pose the 'Science' task, a teacher could demonstrate a toy car rolling down a ramp and ask students to investigate how the angle of the ramp would affect the distance the car moved. As part of the investigation, the students would be attempting to construct an explanation of how and why the angle of the ramp makes the car roll different distances. After the students investigate, the teacher could introduce several science concepts within the context of class discussion of ideas (i.e., potential and kinetic energy, friction, speed, acceleration). Now if the teacher posed an 'Engineering' task, the class might be asked to create a ramp set-up that would allow the toy car to roll a distance of 24 inches. Here, the problem is posed and students are designing and testing ways to get their toy car to roll 24 inches. Although the students are directly experiencing science concepts, the teacher may not even discuss those topics; the focus is on getting a desired outcome.

With both science and engineering, the use of mathematics and technology are vital within the process. In both the above examples, students are measuring angles, measuring distance, recording data, representing the data visually and interpreting those results. The mathematics cannot be separated from either task. In fact, the math can be the language used in both tasks to help students make sense of results; however, the purpose for using math is quite different. When trying to get their toy car to move 24 inches, students are using math when trying to determine how to get a certain result. The data collected can immediately be used to modify the set-up and run more tests; it is based more on problem solving. When investigating the 'Science' task, the math is more structured for students to look for patterns to help develop explanations for why the phenomenon is happening.

For the integration of technology, which many teachers find the most difficult, a teacher would have several options, but the focus should be on using technology as a TOOL not using technology as an OUTCOME. In other words, the technology should only be incorporated if it is being used in an authentic way to augment the activity, not as an add-on to make sure the students get experience with the technology. Using the examples above, for the science task students could record their data into a spreadsheet or other comparable software and use it to display their results. Additionally, after students have investigated with real ramps and cars, they could participate in virtual simulations that would help them make sense of the phenomenon (for further discussion on the rationale of incorporating virtual experiences after real experiences, see Stewart & Vincent, 2013). There are many simulations on-line, and we have found the University of Colorado's PhET simulation site to be valuable not only for the quality but also for the quantity of simulations available (http://phet.colorado.edu/). Additionally, teachers could use a motion detector during this lab so students can see the motion of the toy car in a graph as it rolled down the ramp; in this scenario, the motion is shown visually in the graph and students deal with the movement in a more abstract way (using numbers plotted on x- and y- axes). For the Engineering tasks, because the purpose is different, the same technology might not be appropriate. Students could still be using spreadsheets to organize and display data, but it is unlikely that simulations would be used, as virtual worlds typically assume hypothetical situations involving few external variables (like friction between a car's wheels and the ramp's surface).

So the challenge seems to be how to realistically incorporate all of the STEM disciplines into one activity or lesson. In our experience, it is virtually impossible to do justice to ALL four areas in one activity, especially given the different purposes and outcomes of the Science and Engineering areas. However, that is not to say that teachers cannot use all four areas within units of instruction. If we take the ramp example, a simple way to modify the activity to incorporate all four areas would be to focus on the 'Science' task first. By having students investigate how the ramp angle impacts the distance the car travels, student would be using both mathematics and technology to construct an explanation for the movement of the car. Once students have had an opportunity to run tests, collect data and analyze results, the teacher could lead a discussion on the science concepts of potential and kinetic energy; the teacher could talk with students about the patterns found as they increased the angle of the ramp. Students could use virtual simulations to clarify and deepen their understanding of how the ramp angle (potential energy) impacts the distance the car moves (kinetic energy). Once students have developed an understanding of the science concepts, the teacher could then pose the engineering task of creating a ramp that would cause the toy car to roll 24 inches. Again, students would be using mathematics, but in a different way during this engineering task.

In summary, many teachers might find it difficult to authentically incorporate all four of the STEM disciplines into one activity or task, and for good reason. Because the "S" and the "E" work from different goals, there are few, if any, tasks that would effectively have students do all four disciplines simultaneously. Because mathematics can be the language used to make sense of the physical world, we have found that by structuring tasks from either a 'Science' outcome or an 'Engineering' outcome, we have been much more successful in authentically incorporating mathematics and technology.

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Closing the Fractions Gap with Early Childhood Learners

By Jim Franklin

Mastering fraction standards can be a complex and daunting challenge for elementary school students. In many elementary school classrooms, teachers typically introduce fractions in the first phase of instruction as "How many parts are shaded or unshaded?" Then, they begin to draw fraction number lines, rectangles and circles with shaded areas. Visual and kinesthetic students benefit from using fraction tiles, fraction cubes, blocks, and magnetic fraction strips. Unfortunately, some students use them as "toys," causing loss of instructional time and other behavioral problems.

In the second phase of teaching fractions, many students eventually understand the concept of equivalent fractions. However, they frequently struggle drawing parts in equal proportions when asked to create a visual representation due to weak fine motor skills and require modeling of grade- appropriate work from their teachers. There are some students who need small group instruction and assistive technology, and in some instances, individual instruction is required to achieve mastery of the standard due to attentional issues or learning disability. These problems make the standards of adding and subtracting fractions with the same denominators, different denominators, and mixed numbers even more difficult, but the skills must be introduced and eventually mastered by the students. While making recommendations of how to help students create equal parts beyond the scope of the paper, we acknowledge not all students are adept at it yet when being required to participate in phase 2 and phase 3 learning.

In the third phase of teaching fractions, elementary teachers implement the strategy of folding fraction strips (halves, 3rds, 4ths, 6ths, and 12ths) or fraction triangles (halves, 4ths, 8ths, and 16ths.). These strategies allow students to collaborate with their peers and explore the important fractions' concepts. Another benefit is the requirement for students to follow step-by-step directions to accurately draw or create visual representations. These strategies are effective to a certain point. Yet, many problems can eventually occur because fraction strips and fraction triangles are often damaged through inappropriate handling by the students, and some of the parts are not equivalent. When the fraction strips and fraction rectangles are being made by the students, teachers must exhibit patience with struggling students, flexibility in the pace of their lessons, and willingness to provide additional paper for struggling students.

In the fourth phase of teaching fractions, when teachers introduce finding the sum and difference of fractions, they usually begin by using common denominators for fractions with common and uncommon fraction mixed numbers. At this time, students begin to separate into the group that understands and the other group who does not understand. They often refer to using the visual representation models that are posted on anchor charts or models that the students have created in the first three phases. The concepts of fraction exploration and collaboration among peers are very effective in the understanding the relationship between parts and whole quantities as well as improving the students' ability to explain the steps that were used to find their answers. Unfortunately, students with weak foundational skills struggle to work collaboratively during this phase when they are asked to explain their observations, assist another struggling student, and find a correct answer.

Teachers usually use assessments, work samples, and classroom observations to determine if students have mastered the ability to successfully draw visual representations of fractions. Due to time constraints, some teachers have students who need extended time, but they must move on to standards involving different denominators and mixed numbers. The strategy of finding common denominators can be quite confusing for students who exhibit poor math fact fluency and/ or have a limited understanding of fractions smaller than ¼.

Teachers often rely this four-phase model below to teach fractions:

Phase I	**Phase II**	**Phase III**	**Phase IV**
Visual Representation by drawing parts/whole, number lines, use of blocks, and folding fraction strips and fraction triangles. Students are expected to explain their work.	Implement strategies to have the students find, create, and explain equivalent fractions.	Find the sum and difference of fractions with the same denominators by using models and anchor charts. Students are expected to explain their work.	Find the sum and difference of common and uncommon fractions with different denominators in mixed numbers. Students are expected to explain their work.

Many students struggle to achieve mastery of adding and subtracting mixed number fractions with different denominators because they lack mastery at some point through the progression of the phases. Over the years, teachers have had very limited options to make this transition due to the students struggling with the fractions' math standards and limited resources. Despite students' receiving quality instruction in large and small group settings, receiving instructional accommodations, and using variety of visual representations to learn fractions, many students begin to struggle because they need a fraction manipulative to bridge the gap. Most students understand common fractions, such as ¼, ½, and ¾. However, they begin to struggle when they are required to find the sum and difference of less common fractions with denominators of 3, 6, 8, 12, and 16. Because many students do not have a strong understanding of visual representations of common fractions, they lack the foundation and confidence to master increasingly difficult math standards that involve abstract denominators, such as 5, 7, 9, 11, etc.

The strategies mentioned thus far have received some success in a four phase approach, and they need to continue to be implemented to help students gain a solid foundation to grasp the understanding of fractions because students are expected to progress from the concrete visual representations to the abstract. However, teachers must find additional strategies to bridge the fraction gap and determine why students continue to struggle with learning fractions' standards.

By expanding the phases of instruction from four to six, teachers will be given opportunities to provide additional instruction and slow the pace of instruction to meet the needs of struggling students, but still meet instructional deadlines in their curriculum map. Teachers are encouraged to use their successful strategies in Phases I and II with their students.

In the third phase, by using a number line strategy with Slide-A-Round Math Fractions 12^{ths} and Fraction 16ths manipulatives, students can search for equivalent fractions. These manipulatives will replace the student- made fraction strips, fraction triangles, and other visual representations because the fractional parts are consistent, durable, and cost effective. Students are still encouraged to collaborate with their peers and continue their fraction exploration with the manipulatives. When student use the manipulatives, teachers will find that principle of great teaching is when students learn when they do not know they are being taught. With these manipulatives, students will continually see the importance of the highlighted fraction ½ and many of its equivalents. They will recognize fractions that are greater and less than one- half and not all fractions can be simplified. Students will learn through self- discovery that they can rank the manipulatives from least to greatest and greatest to least.

With the Fractions 12ths manipulative, for example, students will make the connection that the fractions, such as halves, 3rds, 4ths, 6ths, and 12ths, on the manipulative are the same as the parts on their fraction strips. They will also notice that the Fractions 16ths manipulative is an extension of the folding triangles strategy

The fourth phase of teaching fractions to students is because they are introduced how to add and subtract fractions with different denominators. Students often lose sight that there are numbers between the whole numbers. With the Slide-A-Round Fractions 12ths and Fractions 16ths manipulatives, students become aware that fractions are and will always be present between whole numbers and that not all fractions can be reduced to simplest form. On the Fractions 16ths manipulative, for example, students are taught that the lines on the main part should be considered as string that is divided in halves, 4ths, 8th, and 16ths and the numbers on the “slides” are whole numbers. In addition to comparing fractions, students can add and subtract different denominators and reduce their answers without paper and pencil. Please note that it is highly recommended that students master the fractions triangle strategy before using the manipulative.

In the fifth phase of teaching fractions, students will find the sum and difference of fractions that are on the Slide-A-Round Fractions 12ths and Fractions 16ths manipulatives. By using fractions that are common to most students and problems that they have already solved with the manipulative, teachers can continue to increase their students’ confidence by introducing a five- step “Fractions Cheat Sheet” to guide them to solve adding and/ or subtract fraction math problems. After they use the five- step strategy and get their answer, they use their manipulatives to check their work. Most importantly, when students use the manipulatives, they can hold themselves accountable by independently finding their answers and manipulatives give them a resource to check their work if their teacher is unable to immediately assist them. It is just as important for the students to determine if their answers are correct AND wrong. As a result, teachers will be given additional time in class to provide individual and small group instruction for struggling students who are have questions in Phases I-IV rather than being distracted by students who need reassurance that their answers are correct during class.

Students are encouraged to follow the five-step “Fractions Cheat Sheet” to add and subtract fractions with different denominators.

Steps to Solve Fractions:

1. Read the problem: 2/7+ 1/3 =________

2. Rewrite the problem vertically.

   ____
   \+ _______
   ____

3. Find the denominator in the math problem (Hint: The denominator is the bottom number in a fraction.). Then list the multiples and circle the least common multiple (LCM).

   _________ = ____
   Denominator Least Common Multiple (LCM)
   _________ = ____
   Denominator Least Common Multiple (LCM)

4. Put the least common multiple (LCM) under the line next to rewritten problem. (Hint: Think "What times what = new fraction".) Be sure to multiply the same number to the top (numerators) and bottom (denominators) parts of the fractions.

   Rewritten Problem New Fractions with the LCM

   _______ x = _______
   x =
   \+ _______ x = _______
   x =
   _________ __________

5. Add the numerators of the new fractions to get final answer. DO NOT add the denominators.

   ____

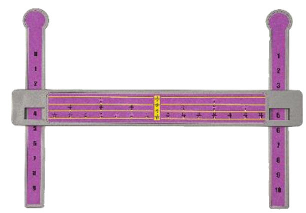
In the sixth phase of teaching fractions, teachers introduce mixed numbers with common and uncommon denominators. It is important to note that these problems will not be found on the Slide-A-Round Fractions 12ths and Slide-A-Round Fractions 16ths manipulatives. At this time, students developed a strong foundation of skills for fractions.

Struggling students can still use their “Fractions Cheat Sheet” to find their answers. However, teachers will eventually encourage their students to become independent of it.

Here is a common addition math problem that involves fractions that can be used with the Slide-A-Round 16^{th} Fractions manipulative:

Problem:

4 1/8 + 3/16 =____



Slide-A-Round 16^{th} Fractions manipulative

For a student to find the answer on the Fractions 16^{th} manipulative, he must complete the following steps:

1. Establish the correct whole numbers in the manipulative's windows by having a 4 in the left window and 5 in the right window.
2. Find the fraction 1/8. Note: The student will recognize that the fractions 1/8 and 2/16 are equivalent fractions.
3. “Drop down” to 2/16 because the next part of the math problem has a denominator of 16
4. Move 3/16 by counting to the right.
5. The final answer is 4 5/16. Because there are not any fractions above 5/16, the student will learn that the fraction is reduced to simplest form.

On the Fractions 12ths manipulative, for example, students are taught that the lines in the main part should be considered as string that is divided in halves, 3rds, 4ths, 6ths, and 12ths, and the numbers on the “slides” are whole numbers. In addition to comparing fractions, students can add and subtract mixed numbers with different denominators and then reduce their answers without paper and pencil. Please note that it is highly recommended that students master the fractions’ strip strategy before using the manipulative.

Here is a common subtraction math problem that can be solved with the Slide-A-Round Fraction 12^{ths} manipulative:

Problem:

4 2/3 – 2/12 =_____

Slide-A-Round 12^{th} Fractions manipulative

For a student to solve the problem with the Fractions 12^{ths} manipulative, he must follow these steps:

1. Establish the correct whole numbers in the manipulative's windows by having a 4 in the left window and 5 in the right window.
2. Find the fraction 2/3. Note: the student will recognize that the fractions 2/3, 4/6, and 8/12 are equivalent fractions.
3. "Drop down" to 8/12 because the next part of the fraction has a denominator of 12
4. Move 2/12 by counting to the left.
5. The unreduced answer is 4 6/12. The student will recognize 4 6/12 can be reduced to 4 ½ for his final answer because of the reduced fractions above it on the fractional number line.

The purpose of the new fractions' manipulatives, which were created based on classroom experience and students' needs, is to bridge the gap from visual representation to the abstract. Here is a new six phase model for teachers to follow when teaching fractions:

Phase I Visual Representation by drawing parts/whole, number lines, use of blocks, and folding fraction strips and fraction triangles. Students are expected to explain their work.	**Phase II** Implement strategies to have the students find, create, and explain up to three equivalent fractions.	**Phase III** Find the sum and difference of fractions with the same denominators by using models and anchor charts. Students will explain their work.
Phase IV Use Slide-A-Round Fractions 12^{ths} and Fractions 16^{ths} manipulatives to bridge visual representation to find the sum and difference of fractions and mixed numbers that include only common fractions. Student will explain their work.	**Phase V** Introduce finding denominators strategy with common fractions by using "Fractions Cheat Sheet." Use Slide-A-Round Fractions 12^{ths} and Fractions 16^{ths} manipulatives to check students' work. Students will explain their work.	**Phase VI** Find answers to common and uncommon mixed number fractions. Students are independent of Slide-A-Round Fractions 12^{ths} and Fractions 16^{ths} manipulatives and "Fractions Cheat Sheet." Students will explain their work.

By breaking down a difficult standard into manageable parts, teachers can determine if their students require additional interventions and/ or need additional practice with a rubric. Teachers can provide their students the "Six Phases of Fractions Rubric" to monitor their progress; they also have the option to conference with the students on a weekly basis or daily basis if necessary. The rubric allows the teacher to group students for collaboration purposes based on their ability levels. Most importantly, by using the rubric and conferencing with their students, teachers will increase students' accountability for their progress and assist them in asking more precise questions, especially students who have difficulty expressing themselves orally due to low tolerance of frustration, limited English proficiency, and/or speech impairment. As a result, instructional time will be more effective and efficient. Refer to the rubric at the end of the article.

It is important to note that Slide-A-Round Math Manipulatives has designed low vision and braille manipulatives that have received positive feedback from a variety of educational organizations. Students who have low vision and are blind have access to these manipulatives to learn fractions. Students with other disabilities can have access to low-tech assistive technology to meet their grade- appropriate math standards on district, standard, and federal assessments as well as have their accommodations met, as stated in their Individualized Education Plans (IEP.)

Fractions are very essential standards for students to master on classroom tests as well as on standardized assessments. Schools must focus on the short- term as well as the long- term because fractions are used in numerous everyday situations. By providing additional strategies and visual aids to support students' instruction in all academic settings, they are given opportunities to become successful and less frustrated. Traditional methods of teaching fractions are very important and must be used on a regular basis, but innovative strategies are necessary to reach all students.

Jim Franklin is an inclusion special education teacher at Elm Street Elementary in Rome, Georgia. He has taught for over 20 years in inclusion, resource, and self-contained settings. Based on the needs of his students, he has invented several math manipulatives to address math standards for all students, including low vision and blind students. He has delivered numerous trainings and demonstrations at school districts and national and state conferences throughout the United States. Visit www.slidearoundmath.com to watch a tutorial video and for additional information on the fraction manipulatives.

CATEGORY	**Initial Level of Mastery**	**Emerging Level of Mastery**	**Progressing Level of Mastery**	**Mastery Level**
	Date	**Date**	**Date**	**Date**
Phase One	Demonstrates the ability to explain and independently draw visual representations of fractions and create fraction trips/triangles	Demonstrates the ability to explain and independently draw visual representations of fractions and create fraction strips/triangles	Demonstrates the ability to explain and independently draw visual representations of fractions and create fraction strips/triangles	Demonstrates the ability to explain and independently draw visual representations of fractions and create fraction strips/triangles
Phase Two	Demonstrates the ability to locate, create, and explain equivalent fractions	Demonstrates the ability to locate, create, and explain equivalent fractions	Demonstrates the ability to locate, create, and explain equivalent fractions	Demonstrates the ability to locate, create, and explain equivalent fractions
Phase Three	Demonstrates the ability to find and explain the sum and/ or difference of fractions with the same denominators by using models/anchor charts	Demonstrates the ability to find and explain the sum and/ or difference of fractions with the same denominators by using models/anchor charts	Demonstrates the ability to find and explain the sum and/ or difference of fractions with the same denominators by using models/anchor charts	Demonstrates the ability to find and explain the sum and/ or difference of fractions with the same denominators by using models/anchor charts
Phase Four	Demonstrates the ability to find and explain the sum and/or difference of common fractions with different denominators with Slide-A-Round manipulatives	Demonstrates the ability to find and explain the sum and/or difference of common fractions with different denominators with Slide-A-Round manipulatives	Demonstrates the ability to find and explain the sum and/or difference of common fractions with different denominators with Slide-A-Round manipulatives	Demonstrates the ability to find and explain the sum and/or difference of common fractions with different denominators with Slide-A-Round manipulatives
Phase Five	Demonstrates the ability to independently find sum and/ or difference of common fractions with Slide-A-Round manipulatives and check work. Understand the five steps on the Fractions “Cheat Sheet”	Demonstrates the ability to independently find sum and/ or difference of common fractions with Slide-A-Round manipulatives and check work. Understand the five steps on the Fractions “Cheat Sheet”	Demonstrates the ability to independently find sum and/ or difference of common fractions with Slide-A-Round manipulatives and check work. Understand the five steps on the Fractions “Cheat Sheet”	Demonstrates the ability to independently find sum and/ or difference of common fractions with Slide-A-Round manipulatives and check work. Understand the five steps on the Fractions “Cheat Sheet”
Phase Six	Demonstrates the ability to find and explain the sum and difference of common and uncommon mixed number fractions independently	Demonstrates the ability to find and explain the sum and difference of common and uncommon mixed number fractions independently	Demonstrates the ability to find and explain the sum and difference of common and uncommon mixed number fractions independently	Demonstrates the ability to find and explain the sum and difference of common and uncommon mixed number fractions independently

Add Think-Pair-Share to Your Teaching Strategies!

By Carol Lucas

The easiest cooperative learning strategy to utilize in the classroom is Think-Pair-Share (McTighe and Lymann, 1988). The teacher poses a problem to the students and asks them to think quietly as to how to solve the problem. After allowing adequate time for the students individually to come up with a plan, students are asked to pair with a partner to discuss and compare their ideas. During this time, the teacher walks around and listens to the partners to gauge what strategies are being suggested. The partners should agree on a strategy and carry it out to solve the problem. Finally, various partners are asked to share their solutions with the class as a whole.

Allowing students to have time to think seems like a basic idea that teachers would naturally use. However, research has shown that the wait time when a teacher asks a question is less than one second (Rowe, 1978). When wait-time was increased to three seconds or more student confidence in their ability was increased as was creativity. The Think-Pair-Share strategy has built in time for students to think about the problem and also holds each student in the class responsible for having an idea to share with their partner.

Communication is an important process standard of learning mathematics (NCTM, 2000). Providing students the opportunity to "talk math" is an activity that needs to be incorporated into all mathematics classrooms. The pairing part of this cooperative learning strategy requires students to be able to discuss their ideas and use mathematical terminology to convey their plan for the problem's solution.

The final component of the strategy is the sharing of solutions with the whole class. Students are once again asked to communicate mathematically. Since it is likely that different groups solved the problem in different ways, the class members will be exposed to multiple solutions and views of the same problem.

In practice, the strategy must be utilized with problems that require more than a simple immediate response. The problem posed needs to be one that requires thinking and has more than one approach for its solution. Here are some examples of problems that could be used.

Explain how you would add 57 + 49 mentally. Students might use compensation (60 + 46 for example). They might add left to right (90 + 16) or they might have other techniques.

Create a story where you would need to compare ⅔ and ¾. Students might consider the amount of pizza two people have eaten, time passing on a clock, or multiple other scenarios that would involve comparing these two fractions.

You may choose any of 5 pizza toppings to add to a cheese pizza: beef, pepperoni, bacon, mushrooms, or olives. How many choices are there? For a problem like this, students might list all of the possibilities, look for a pattern, or find other ways to answer the question.

After experiencing the use of the Think-Pair-Share strategy multiple times during one semester in a math class taught by me, my students were asked to respond anonymously as to whether they liked or disliked the strategy. They were also asked to explain their reasons. The responses were very positive, and the following comments were typical. For example, one student commented, "I really like think pair share. I love that it gives me a chance to think without someone shouting out the answer, since students think at different rates. I also like that we are able to have one-on-one discussions and have a chance to ask questions without the whole class listening. The share part at the end it a great way to make sure we are grasping the concept and to clear up any confusion some might have had during the "pair" portion of this strategy.

Think-Pair-Share promotes the classroom as a learning community where student's thinking is valued individually while at the same time students are helping one another as they share their ideas. The ease of utilizing this strategy, the benefits of time to think and communicate about mathematics, and the positive feedback from students who have experienced this strategy are all arguments for mathematics teachers to incorporate this strategy in their lesson presentations.

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Are You a Square or a Rectangle? Exploring Relationships

By Denice Hurlbut and Leticia Martin

Finding correlations between sets of data lays the foundation for future algebraic practices. Chambers states that:

> Exploring the relationship between two variables prepares the student for the study of functions, which in turn provides the basis for much of the study of algebra. Students need concrete experiences in meaningful contexts to construct an understanding of the interaction between and among variables. (n.d.)

"Are you a Square or a Rectangle?" is an activity focused on using algebraic thinking to identify correlations between two sets of data and select and create visual representations, such as tables or graphs, to solve a real-world mathematical problem. This activity engages students in Oklahoma Academic Standard for Mathematics 6.A.1 as they recognize and represent correlations between the two sets of data. The lesson also includes a component of measuring in a real-world environment, and best practices for obtaining accurate measurements of an object (in this case a person) that does not have a flat surface to measure along. The activity is designed for use with sixth-grade students, but is suitable for students in other grades.

Within the time constraints of a class, the activity permits significant and needed mathematical learning concerning three topics: correlations, appropriate representations of data, and measurement of real-world objects. The activity begins with a discussion about whether or not a correlation between a person's height and arm span exists, followed by three activities: accurately measuring height and wingspan, deciding appropriate visual representations to illustrate the relationship between heights and wingspan, and exploring and visualizing correlation.

Opening Discussion

The discussion is begun by asking students if they think they are a square or a rectangle, and having a conversation about what that might mean. This conversation is also an opportunity to incorporate the vocabulary words *inscribe* and *circumscribe,* helping students to understand that the question we are really exploring is *Would a square or rectangle best circumscribe you?* This conversation allows students to apply their knowledge of dimensional relationships in geometric figures to a non-standard shape - the human body. Ultimately, the question that students will be answering is *Do you think there is a correlation between a person's arm span and a person's height?* When this activity was conducted with a group of pre-service teachers, this question lead to a discussion of the meaning of the word correlation, specifically, does a correlation mean that the wingspan and height are exactly the same length? This presented an opportunity to talk about the word correlation and what it means to find a relationship between two sets of data. Ultimately, the group predicted unanimously that a correlation exists between a person's height and wingspan.

Activity 1: Accurately measuring height and wingspan

Next, students are instructed to work with a partner in using a tape-measure to find one-another's height and wingspan. This task represents Oklahoma Academic Standard for Mathematics 6.GM.3 by allowing students to choose appropriate units of measurement. This part of the activity provides an opportunity to lead in a conversation about best practices for measuring the length of non-linear objects. We recommend providing a demonstration in which a student is measured standing against the wall, or lying on the floor. Students should be instructed to hold their tape measures taut and straight. Wingtip should be measured from fingertip to fingertip, and make sure the students know to take this measurement across their partners' backs and not across the front of the chest. Also, address the impact that shoes, hats, and posture can have in finding the correct height of a person. Finally, include a discussion about how students can proceed if their tape measure is shorter than the person they are measuring. We found that, even with pre-service teachers, many of these practices for measuring were not being used and that a deeper conversation and demonstration ahead of time would have been very helpful.

As the students collect their measurements, provide a means for them to consolidate them in a way that they are accessible to the whole class. We used a spreadsheet projected onto the whiteboard. If the students have access to netbooks or tablets, the data could be entered into a shared spreadsheet. It could also be written into a table on the board. Return to the question of whether or not the students in the class are squares or rectangles, based on their measurements.

Activity 2: Deciding appropriate visual representation to illustrate the relationship between heights and wingspan

Next students are instructed to create a visual representation of the whole class's data, which implements Oklahoma Academic Standard for Mathematics 6.D.1 on displaying and analyzing data. Students continue to work with their partners, using poster board or butcher paper and markers. Be sure the students understand that their representations must include all the data - heights and wingspans for all members of the class, but allow them to determine for themselves what that will look like. In our group of pre-service teachers, data representations included double bar graphs, line charts, and scatter plots. As students work on their charts and graphs, take the time to ask each group why they chose the type of visual representation that they did. This presents an opportunity to discuss the purpose of the visual representation, and assess if students are looking for correlations in the data.

Once all students have finished, groups are invited to share their visual representations with the rest of the class. It is not necessary for all groups to share, especially when several groups have created very similar visual representations, but try to focus on having any variations shared with the class to allow for a discussion of what can be seen in each representation. Ask students if they could see a correlation in the data once they created a visual representation.

Activity 3: Exploring and visualizing correlation

The final stage of this activity, is inviting the students to create a human scatter plot representing the class's data. To do this, you will need a large section of cleared floor. We had the advantage of having white boards on two adjacent walls and used these as our axes, running painter's tape along the floor and using the floor tile as our grid. Have the class decide what the labels for the x and y axes should be, and what intervals should be numbered off on the board. You might have students refer to their own visual representations for this. With our group, the initial response was to have the class's data points used as the intervals, which provided for a discussion about the need for consistent intervals in our labels. Once the axes were labeled and marked off, the students each find the intersection of their own height and wingspan on the scatterplot, and stand on that point. Then, the students at the low and high ends of the scatterplot are asked to each hold the ends of the same piece of ribbon or yarn, creating a trend line. Have each student assess their proximity to the trend line by having them see if they can also reach the ribbon or yarn. It will be difficult for some of the students to see the big picture of the trend represented in their scatterplot, due to clustering. You might tag students to take a step out get a better look in shifts, or take a picture from a high angle to put on the board.

If you wish to add a discussion of outliers, this can be done by adding Michael Phelps (height: 193 cm; wingspan: 201cm) and/or Russel Westbrook (height: 191 cm; wingspan: 213 cm) to your data set, and adding yourself to the scatterplot in their position to demonstrate that their wingspan and height relationship do not follow the same trend that the class's does by showing how far from the ribbon or string they are. Be sure to emphasize that these are two specific examples of outliers and that not all professional athletes will have a significantly longer wingspan than height.

Once students have returned to their seats, close the activity with a broader discussion of what this correlation within the class means. Ask students what assumptions they could make about someone who is not in your class, based on the activity. Ask if they can be 100% certain that they would be correct?

In the "Are you a Square or a Rectangle" activity, students are presented with opportunities to review and implement best practices for measuring, a skill not used often by most youth. They can accurately gather, represent, and interpret data, and visualize a correlation between data points. This activity prepares them for further algebraic thinking as they seek and recognize patterns and become deeply familiar with how patterns are represented on an x, y axis.

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Gradients, Vector Fields, and Extrema in MatLab

By Brent Strunk

The ability to move conversations from computation to concept is a strong argument for using a computer algebra system such as MatLab in class. The essential nature of technology as an aid to developing understanding is stated in the National Council of Teachers of Mathematics technology principle (NCTM, 2017). Arc length, parametric curves, projectile motion, and vector fields are all topics in which students can find themselves bogged down with detailed computations. Removing computation as a barrier allowed students to dig into not only these topics, but other topics that arose naturally, such as chaos theory and extrema. This happened by allowing the student to use the computer to process and acquire information while allowing the student to engage in deeper reasoning, as outlined in (Devlin, 1997). The goal of relieving students from "tedious calculations" in order to “have a stronger focus on topics” is a common goal among educators using technology (Monaghan, 2007).

Students are exposed to MatLab as part of the first semester of Calculus, allowing students in the second semester to build on that experience. Once the basic MatLab skills are reviewed, the students can start to use the computer to push past computation to the underlying concepts, and then extend those concepts further. This review helps mitigate some of the loss of focus due to the need to explain technical operation, as mentioned in (Monaghan, 2007).

What follows are examples of MatLab assignments that were used to help illustrate the concepts behind the topics covered in Calculus.

Gradients and Extrema

The jump from $\mathbb{R}^2$ to $\mathbb{R}^3$ is one of the toughest jumps in the Calculus sequence. Taking the understanding students have built up in the 2 dimensional setting of a first Calculus course and lifting it to 3 dimensions is challenging for the student and the teacher.

The hunt for extrema is an example of the challenge for all involved. Finding critical points in the 3 dimensional setting is similar to the process in 2 dimensional settings, and the definitions for extrema learned in the 2 dimensional case lifts naturally to the 3rd dimension. However, classifying the critical points is a different process entirely. In addition, the standard techniques generally used can fail to classify a point leaving the students a bit disappointed with the standard method.

One of the final MatLab assignments focused on computing gradients, computing a vector field using the gradient, and the gradient’s relationship with the graph of a multivariate function. This assignment gave the students a chance to examine the First Derivative test in multiple dimensions.

Vector fields are formally introduced in the third semester of a Calculus sequence, but thanks to the ability to easily generate a vector field in MatLab we could preview the topic as part of the gradient conversation. This illustrated the changing magnitude and direction of the gradient vectors. In addition, we could leverage the graphing abilities of MatLab to illustrate gradients as “vectors tangent” to the surface of the graph of the function.

Students were provided a worksheet that introduced the basic commands needed in MatLab and gave examples to work through. Students were allowed to work as a group in class and encouraged to compare findings. Students were required to work through finding the gradient for 3 multivariate functions, plotting the vector fields derived from those gradients, interpreting the vector fields, and plotting the surfaces determined by the functions.

Students were first asked to compute the gradients of a set of curves, starting with the example of $z = x^2 + y^2$.

```
Command Window
>> syms x y
>> gradient(x^2+y^2)

ans =

 2*x
 2*y

>>
```

The curve $z = x^2 + y^2$ was chosen due to the ease of computation by hand of the gradient, and the clear minimum value at (0,0). The students were asked to use MatLab to compute the gradient of the more complicated curves, $z = x^3 + y^3 - 4xy$, $z = x^2y^2e^{x-y}$, and $z = \frac{x^2y^2}{\sqrt{x^2+y^2+1}}$. The gradients are of escalating difficulty to compute by hand to help build confidence in using MatLab as a tool.

Pushing it further: "Gradient Test?"

Students were then introduced to the quiver and mesh grid commands necessary to compute and plot the gradient of the function. Once students had computed the gradient and vector fields for a few functions, students were asked to consider the question of whether the gradient can help students find maximum and minimum points for functions in $\mathbb{R}^3$. Students had learned prior to the assignment the properties of the gradient, especially two properties that were important for this conversation: the gradient indicates the direction of greatest increase along a function and the gradient is a zero vector or does not exist at local maximum and minimum values.

To introduce the question students were shown how to use the mesh grid and quiver command to compute the gradient and vector field for the function $z = x^2 + y^2$.

Command Window

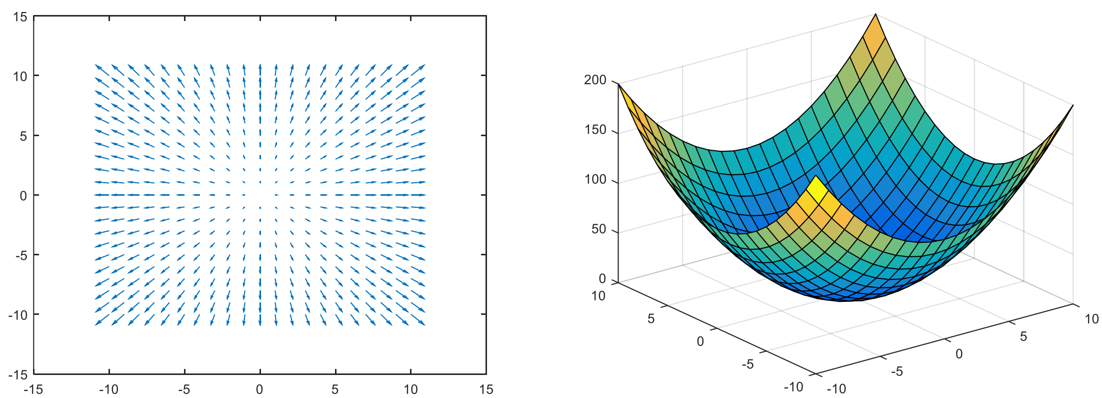
```
>> clear
>> [x,y]=meshgrid(-10:1:10)

x =

   -10    -9    -8    -7    -6    -5    -4
```

```
>> quiver(x,y,2*x,2*y)
>>
```

This function had a clear minimum value at (0,0). The vector field indicated this with a zero vector gradient at (0,0) and gradients that pointed away from the origin in all directions.

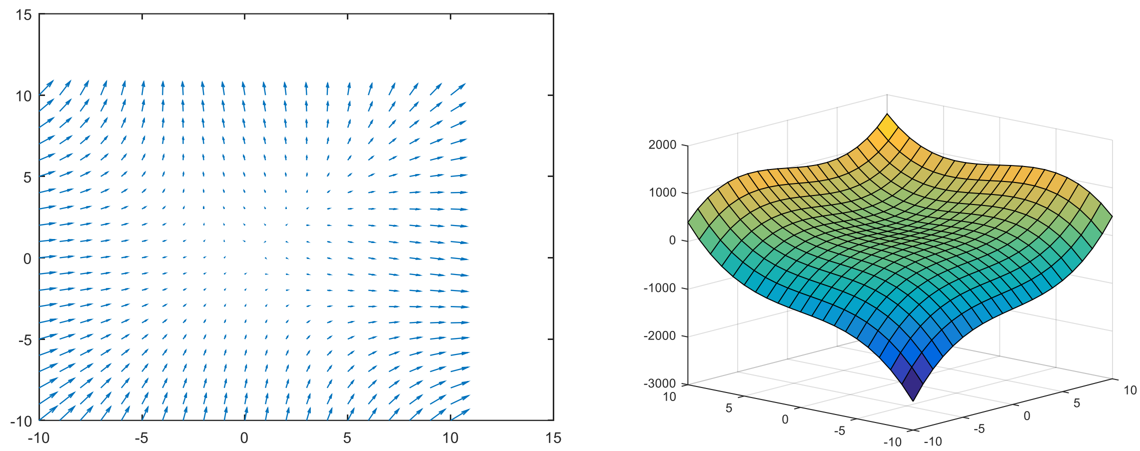


Vector field derived from gradient of $z = x^2 + y^2$ and graph of the surface created using MatLab

Conversation immediately turned towards whether this would work in general. Could the standard method for classifying extrema be replaced by the more intuitive vector field we just built? As an educator, this line of conversation was especially exciting. Vector fields in general would not be covered for another semester. Because of the ease of construction MatLab afforded us, students had not only already accepted vector fields a semester early, but they were trying to push on to find an application for the concept.

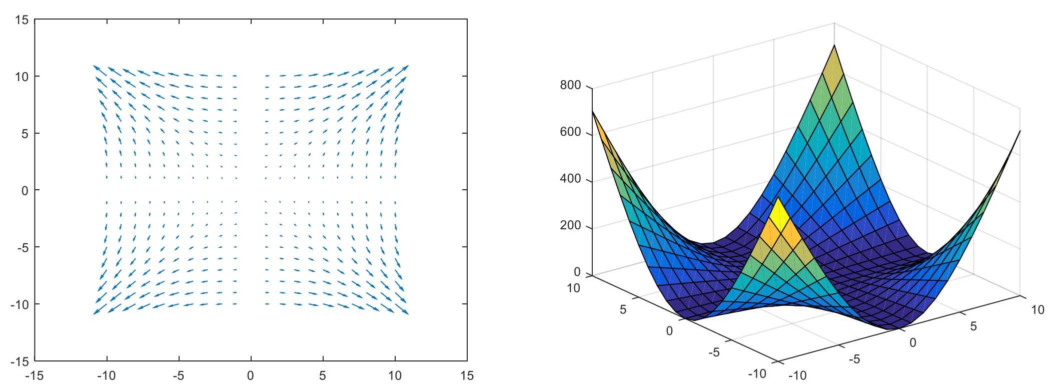
The students were asked to then work in groups using MatLab to compute gradients and vector fields related to three other curves. They were required to turn in their vector fields along with an interpretation of what they saw in the graph, focusing on identifying potential extrema and saddle points. This was followed by asking the students to compute the actual graphs of the surfaces to verify their interpretations.

The first vector field the students were asked to compute was $z = x^3 + y^3 - 4xy$, which has a clear saddle point using the same reasoning as before.



Vector field derived from gradient of $z = x^3 + y^3 - 4xy$, along with the graph of the surface created using MatLab

Two further examples were selected to illustrate some of the difficulties that may occur: $z = x^2y^2e^{x-y}$ and $z = \frac{x^2y^2}{\sqrt{x^2+y^2+1}}$. The first, $z = x^2y^2e^{x-y}$, had very small gradient magnitudes making the graph difficult to interpret. In contrast, $z = \frac{x^2y^2}{\sqrt{x^2+y^2+1}}$ had no local minimum, though the vector field is easier to interpret and seems to imply a minimum at (0,0).



Vector field derived from gradient of $z = \frac{x^2y^2}{\sqrt{x^2+y^2+1}}$ along with the graph of the surface created using MatLab

Even as these examples illustrated potential pitfalls to using a vector field to find extrema, conversations in class turned towards ways to "fix" these problems. For the small gradients, students experimented with zooming, but also scalar multiplication as a way to make the graphs easier to read. With ambiguous vector fields, students started experimenting with more data points, discussing how much data would be "enough" to make a conclusion. These examples of vector fields together with the graphs of the surfaces helped students understand the shortcomings of trying to use vector fields to find extrema and illustrated for the students parallels between single dimensional calculus and multidimensional calculus.

Summary

Integrating computer algebra systems and mathematical software is a noble goal for the Calculus sequence. Current students will need familiarity with computer systems moving forward as these programs become more common both in the classroom and in industry. Utilizing this technology to help students move past computation and towards concepts can yield rewards for the both the educator and the students.

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Professional Resources

Accessible Mathematics: 10 Instructional Shifts That Raise Student Achievement, Steven Leinwand, 2009, 113pp, $17.00, paperback, ISBN 978-0-325-02656-5, Heinemann.

Teachers no longer have to surrender to the rigorous demands of public education's prescribed curriculums, pacing schedules, and their own guilt complexes, which often leave them feeling like ineffective mathematics teachers. In his book, *Accessible Mathematics: 10 Instructional Shifts That Raise Student Achievement,* Steven Leinwand provides teachers useful strategies to improve their mathematics instruction immediately with a few powerful shifts in their existing pedagogy including making meaningful, incremental changes to how they plan, implement, and assess their daily mathematics instruction.

Within each chapter, Leinwand highlights an instructional shift in a digestible language and format that teachers can easily absorb, process, and implement into their existing instructional routine. He usually begins the chapter with a real-world vignette describing a typical or sometimes atypical day in the mathematics classroom. Then in a very systematic manner, he dissects the vignette into essential skills underlying the specific lesson, explaining why and how teachers can connect them to the larger mathematics concept under study. Finally, he summarizes each chapter with a special section titled "so what we see in an effective mathematics classroom" of useful strategies in teacher-friendly bullets.

Throughout the book, Leinwand makes important pedagogical connections between mathematics and other content areas. He gives credit to the success in language arts with the use of the reading/writing workshop model in chapter three: "It's Not Hard to Figure Out Why Reading Works Better than Math." Leinwand specifically points out classroom discourse as important for the mathematics classroom. Another connection he makes between mathematics and other content areas of instruction is through art in chapter four: "*Picture It, Draw it.*" In this chapter, he relates "the power of visualizing mathematical entities," (Leinwand, 2009, p. 19) that often facilitates student comprehension of the problem's context. In chapter five: "Language-Rich Classes," Leinwand makes the direct correlation between comprehensible mathematics and a language-rich environment for all students, and especially for the increasing number of English Language Learners. Then Leinwand couples the language-rich environment with the importance of effective discourse that in tandem provide "the best vehicles for helping English Language learners to prosper," (Leinwand, 2009, p. 28).

The ultimate goal of his book is to empower teachers with effective strategies and improve students' mathematics achievement by helping them build a solid foundation of fundamental mathematics skills through more effective instruction. Beginning in chapter one: "*Ready, Set, Review,"* Leinwand emphasizes the importance "of one of the most effective strategies for fostering mastery and retention of critical mathematics skills and concepts is daily cumulative review," (Leinwand, 2009, p. 6). Through the daily review a teacher can reteach, reinforce, and assess student understanding of multiple mathematics-related skills. Again, in chapter six, "*Building Number Sense,"* he demonstrates through real-world classroom examples of how students often lack a deep understanding of number sense which then, impacts their ability to successfully apply and understand other related mathematics concepts throughout the school year.

In several chapters, Leinwand points out why mathematics should include problems based upon student interest and the world around them. In chapter seven, "*Milking the Data;"* chapter eight*, "How Big, How Far, How Much;"* and chapter ten*, "Putting It All in Context,"* Leinwand explains how teachers can draw connections directly between mathematics skills and concepts to the students' world in order to arouse their interest and actively participate in the mathematics to solve even the most fundamental and sometimes mundane of textbook problems. By incorporating within classroom discussions questions like "How big? How far? How much? and How many? teachers can provide students with interesting and thought-provoking mathematical tasks" (Leinwand, 2009, p.52). To further develop and improve student understanding, interest, and achievement in mathematics, a teacher could employ the questions from chapter eleven, *"Just Ask Them 'Why?'"* Where Leinwand labels them as "classroom mantras: Why? How do you know?" and "Can you explain?" (Leinwand, 2009, p. 69). Therefore, when a teacher brings mathematics into the students' world, he/she not only makes the mathematics applicable to them, but also captures their interest in finding the solutions, thus raising student achievement.

Unlike most of the other chapters in the book where teachers make specific instructional shifts in their mathematics instruction to raise student achievement, chapter nine, " *Just Don't Do It!"* and chapter twelve, "*Punting Is Simply No longer Acceptable,"* deal with teachers, administrators, and the community somewhat independently from the students. In chapter nine Leinwand makes the argument that teachers need to "minimize what is no longer important, and teach what is important when it is important to do so," (Leinwand, 2009, p. 54). Leinwand uses the following standard when making the same determination about his own instruction, "Do I really care whether my children or grandchildren know and can do this?" (Leinwand, 2009, p. 55). In chapter twelve, Leinwand puts the onus on teachers to do the math themselves before asking their students to do so. It is perhaps this shift in particular that gives teacher first-hand knowledge of how to effectively make the other shifts in their instruction that ultimately raise student achievement.

Finally, in chapter thirteen, *“We All Have a Role to Play and Teachers Can’t Do it Alone,”* delegates the burden of educational expectations and responsibilities beyond the classroom soliciting support from the districts, communities, and government officials to raise student mathematics achievement. Through professional collaboration and community interaction, more and more schools can support teachers to make the necessary instructional shifts outlined in the book and better serve their students who can then themselves raise their mathematics achievement.

Because of Leinwand’s book, teachers now have a no-nonsense, reliable resource with ten very effective ways to shift their mathematics instruction in order to improve their students’ mathematics understanding, and help them raise their mathematics achievement.

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Beyond Good Teaching: Advancing Mathematics Education for ELLs. Sylvia Celedon-Pattichis, Nora G. Ramirez, 2012, 236pp, $32.26 paperback, ISBN-13: 978-0873536882. National Council of Teachers of Mathematics.

In the book Beyond Good Teaching: Advancing Mathematics Education for ELLs, the editors discuss strategies to help engage students who are learning English as a second language. Celedon-Pattichis and Ramirez start off recording personal stories of students and teachers who were labeled ELLs while growing up in the public school system. This sheds light on common experiences and issues that arise when working with ELLs. These testimonials attempt to raise awareness of students’ personal feelings and experiences as they work to learn a new language and succeed in school, along with how teachers can better create an environment for success.

As the book progresses, different authors discuss the stages of second language development. There are three basic stages: beginning, intermediate, and advanced. The authors provide information about the characteristics of learners in each stage, along with common student actions and appropriate teacher actions that accompany each stage. Next, readers are provided with a unique perspective on learning a second language form an Apache woman. Rea Goklish details her thought process as she progresses with learning the English language. Her explanation of how ELLs have to translate and interpret what they hear and say emphasizes the importance of wait time in the classroom.

Celedon-Pattichis and Ramirez continue by defining four elements of an effective mathematics community. They discuss how teachers need to have high expectations for all students; take time to listen, observe, and learn; find a balance between an individualistic and collectivistic value system in the classroom; and affirm the cultures of ELLs in the classroom.

The book devotes multiple chapters to case studies from the elementary and secondary levels that demonstrate effective methods for teaching mathematics to and assessing ELLs. The cases provide concrete examples of teachers uncovering and addressing common misconceptions and errors that ELLs encounter. Suggestions are given for teachers to support language development and math achievement while not compromising high expectations and cognitive demand of tasks. It also provides examples of how teachers can use the Mathematical Practices to establish high expectations for all students. In terms of assessment, these case studies emphasize the need to distinguish between students' understanding of the language of the problem and of the mathematics of the problem.

Additionally, the authors offer a variety of suggestions for getting parents involved in their child's learning. It discusses how teachers can open up the classroom to parents to show a respect and appreciation for the way they learned to do math while also being able to demonstrate to them how their children are currently learning math.

A significant chapter of the book provides information and resources that assist teachers in lesson planning while specifically keeping ELLs in mind. It also draws attention to the need for teachers to reflect on lessons to evaluate their effectiveness for all learners in their class. It provides specific questions and teacher actions to help teachers ensure they are developing appropriate lessons to adequately support ELLs.

An additional section of the book highlights the connection between language and mathematics. Teachers must take into consideration the language demands inherent in math instruction. It includes a planning tool to help teachers meet the 5 language modalities (reading, writing, speaking, listening, and representing) from beginning to end of math lessons. Along those same lines, the book focuses on using word problems with ELLs. ELLs often have two main difficulties when solving word problems: lexical (vocabulary in the problem) and sentence structure. It also provides guiding questions for teachers to use as they structure word problems to use with ELLs.

Celedon-Pattichis and Ramirez present four different lenses to analyze the effectiveness of math lessons. There are guiding questions for each lens to make sure that a teacher has given ample thought and reflection of the lesson to ensure it is accessible for all learners. These lenses

allow teachers to take into account the culture, background knowledge, and participation levels of students to make their teaching more effective.
The book concludes with additional resources for further professional development, including videos, PowerPoint presentations, and other online resources.

Beyond Good Teaching: Advancing Mathematics Education for ELLs has numerous strengths as a teacher resource, particularly for those who are just beginning their teaching career and teaching ELLs for the first time. It provides suggestions for making the contexts of problems culturally sensitive and relevant to prevent the context from hindering the mathematics. Video clips from some case studies are available that showcase a variety of strategies, making them more tangible and accessible to teachers. Lesson planning ideas are offered to help teachers prepare to better meet the needs of ELLs. The book has a focus on creating an environment that fosters learning for all students and how to differentiate for ELLs without compromising high expectations. Another benefit is the suggested ways to get parents meaningfully involved in children's learning. Finally, the book come with an accompanying website with an abundance of additional resources.

While Celedon-Pattichis and Ramirez's book is a strong resource, it also has a few limitations. First, the book has a focus on Spanish-speaking ELLs and does not provide much information for teaching students who speak other languages. The book contains very little information on teaching emergent/beginning English speakers. Lastly, it focuses on suggestions for entry-level teachers, while lacking the necessary expansion of strategies for more seasoned teachers.

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All manuscripts will be sent out for blind review to at least 2 external reviewers and one in-house reviewer. Manuscripts will be returned to authors with a summary of reviewers' comments.

Teacher to Teacher**:** Submit descriptions of teaching activities that have helped students learn an essential mathematics skill, concept, strategy, or attitude. Submissions should be no longer than 1500 words, typed and double-spaced, and follow the following format:

- Title (if adapting from another source, cite reference and provide a bibliography).
- Purpose of activity, including the mathematics skill, concept, or strategy students will learn.
- Description of activity with examples, questions, responses. Please provide enough detail so someone else can implement the activity.
- How activity was evaluated to know if purpose was achieved.

Teacher Research**:** Submit manuscripts that describe research or inquiry conducted in classrooms. Submissions should be 1000-2000 words, typed and double-spaced following guidelines of the APA, 5th Edition, and follow this format:

- Description of the question or issue guiding the research/inquiry, including a short review of pertinent literature.
- Description of who participated in the study, what the sources of data were, how the data were gathered and examined.
- Description of the findings and conclusions from the research/inquiry.

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- Introduce and describe the study or studies, including purpose, information about who participated and in the study, how and what type of data was gathered, and the findings or conclusions.
- Discuss the implications of the study or studies for classroom teachers. The implications could include a discussion of what the study told us about mathematics learners and mathematics learning and/or what the study implies teachers should do to support learning.

Professional Resources**:** Submit reviews of professional resources of interest to teachers or mathematics curriculum specialists. Resources reviewed could include books for teachers, books for children, curriculum packages, computer programs or other technology, or games for children. Submissions should be 500-1000 words, typed and double-spaced following guidelines of the APA, 5th Edition, and following this format:

- Title, author, publisher of the resource, year published; number of pages, cost.
- Short description of the resource.
- Critical review of the resource, including strengths and weaknesses.
- Short discussion of how the resource might be useful to a teacher.